



---

Navigation and Ancillary Information Facility

# Derived Quantities

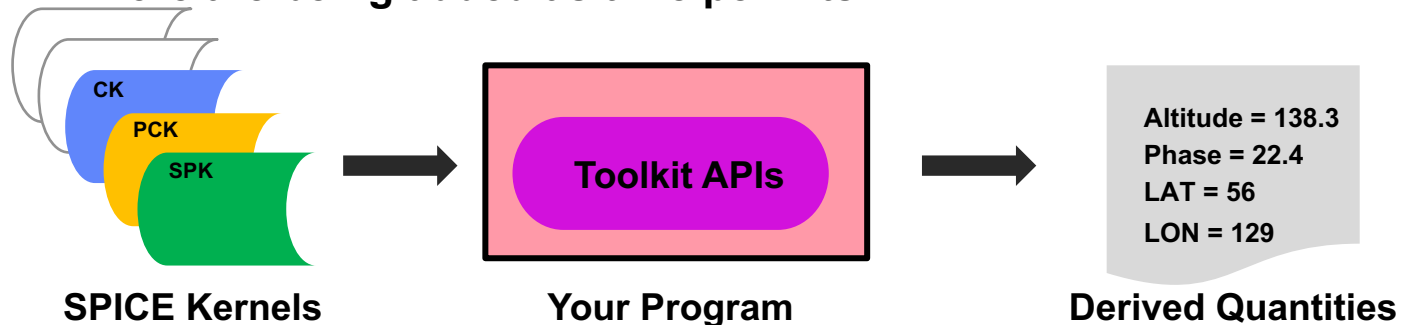
October 2022



# What are Derived Quantities?

Navigation and Ancillary Information Facility

- **Derived quantities, what we often call “observation geometry,” are produced using data from SPICE kernels and Toolkit software.**
  - **These are the primary reason that SPICE exists!**
- **The SPICE Toolkit contains many routines that assist with the computations of derived quantities.**
  - Some are fairly low level, some are quite high level.
  - More are being added as time permits.



- **Examples follow on the next several pages.**



# High-level Geometric Computations

---

Navigation and Ancillary Information Facility

- **Geometric Parameter**
  - Determine a geometric quantity or condition at a specified time.
- **Geometry Finder (GF)**
  - Find times, or time spans, when a specified “geometric event” occurs, or when a specified “geometric condition” exists.
    - » This is such a large topic that a separate tutorial (“geometry\_finder”) has been written for it.

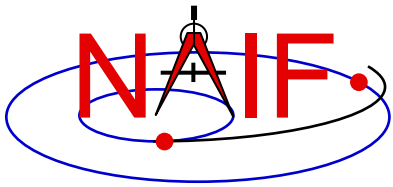


# Examples of Geometric Parameters - 1

## Navigation and Ancillary Information Facility

- **Illumination angles (phase, incidence, emission)**
  - ILLUMF, ILLUMG, ILUMIN\*
- **Sub-solar point**
  - SUBSLR\*
- **Sub-observer point**
  - SUBPNT\*
- **Surface intercept point**
  - SINCPT\*, DSKXV, DSKXSI
- **Tangent point**
  - TANGPT
- **Phase angle between body centers**
  - PHASEQ
- **Angular separation between two objects**
  - TRGSEP

\* These routines supercede the now deprecated routines ILLUM, SUBSOL, SUBPT and SRFXP



# Examples of Geometric Parameters - 2

---

Navigation and Ancillary Information Facility

- **Longitude of the sun (Ls), an indicator of season**
  - LSPCN
- **Limb points on an ellipsoid or DSK**
  - LIMBPT
- **Terminator points on an ellipsoid or DSK**
  - TERMPT
- **Surface points at specified longitude, latitude coordinates**
  - LATSRF
- **Outward surface normal on extended object**
  - SRFNRM
- **Azimuth and elevation of a target as seen from a constant position observer**
  - AZLCPO



# Examples of Geometric Conditions

---

Navigation and Ancillary Information Facility

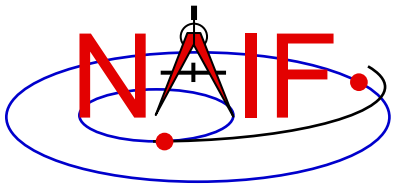
- **Ray in field-of-view?**
  - FOVRAY
- **Ephemeris object within field-of-view?**
  - FOVTRG
- **Determine occultation condition**
  - OCCULT



# Examples of Geometric Searches

Navigation and Ancillary Information Facility

- **Find times when:**
  - ray is in field-of-view
    - » GFRFOV
  - ephemeris object is within field-of-view
    - » GFTFOV
  - object is in occultation or transit
    - » GFOCLT
  - object is at periapse
    - » GFDIST
  - latitude and longitude are in specified ranges
    - » GFPOSC
  - solar incidence angle is below a specified limit
    - » GFILUM
- **For more GF functionality, see the GF tutorial.**



# Geometry Related to Objects

Navigation and Ancillary Information Facility

## Function

## Routine

- **Ellipsoids**

- nearest point
- surface ray intercept
- surface normal
- point on surface
- limb
- slice with a plane
- altitude of ray w.r.t. to ellipsoid

- NEARPT, DNEARP
- SURFPT
- SURFNM, EDNMPT
- EDPNT
- EDLIMB
- INELPL
- NPEDLN

- **Planes**

- intersect ray and plane

- INRYPL

- **Ellipses**

- project onto a plane
- find semi-axes of an ellipse

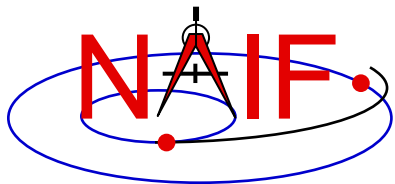
- PJELPL
- SAELGV

- **Lines**

- nearest point on line or segment

- NPLNPT, NPSGPT





# Position Coordinate Transformations

Navigation and Ancillary Information Facility

## Coordinate Transformation

## Routine

- Transform state vector between two coordinate systems

– XFMSTA

General purpose  
state transformation API  
(except Az-El)

- Latitudinal to/from Rectangular

– LATREC  
RECLAT

- Planetographic to/from Rectangular

– PGRREC  
RECPGR

- R.A. Dec to/from Rectangular

– RADREC  
RECRAD

- Geodetic to/from Rectangular

– GEOREC  
RECCEO

- Cylindrical to/from Rectangular

– CYLREC  
RECCYL

- Spherical to/from Rectangular

– SPHREC  
RECSPH

- Azimuth-Elevation to/from Rectangular

– AZLREC  
RECAZL

Single purpose APIs



# Velocity Coordinate Transformations

Navigation and Ancillary Information Facility

## Coordinate Transformation

## Routine

- Transform state vector between two coordinate systems

– XFMSTA

General purpose state transformation API (except Az-El)

- Latitudinal to/from Rectangular

– DRDLAT  
DLATDR

- Planetographic to/from Rectangular

– DRDPGR  
DPGRDR

- R.A. Dec to/from Rectangular

– DRDLAT  
DLATDR

- Geodetic to/from Rectangular

– DRDGEO  
DGEODR

- Cylindrical to/from Rectangular

– DRDCYL  
DCYLDR

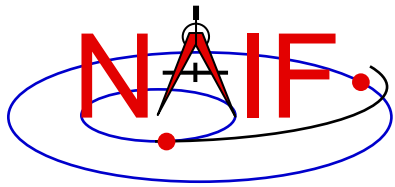
- Spherical to/from Rectangular

– DRDSPH  
DSPHDR

- Azimuth-Elevation to/from Rectangular

– DRDAZL  
DAZLDR

Single purpose APIs



# Examples of Velocity Coordinate Transformations

Navigation and Ancillary Information Facility

**This example is for rectangular to spherical**

- **Using full state vector transformation API**

```
CALL SPKEZR ( TARG, ET, REF, CORR, OBS, STATE, LT )
```

```
CALL XFMSTA ( STATE, 'RECTANGULAR', 'SPHERICAL', ' ', OUTSTATE )
```

- **Using velocity-only (Jacobian) APIs**

- Transform velocities from rectangular to spherical coordinates using the SPICE Jacobian matrix routines. The SPICE calls that implement this computation are:

```
CALL SPKEZR ( TARG, ET, REF, CORR, OBS, STATE, LT )
```

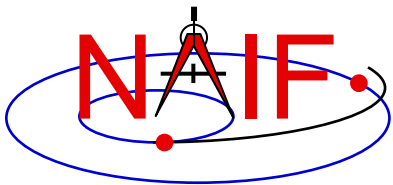
```
CALL DSPHDR ( STATE(1), STATE(2), STATE(3), JACOBI )
```

```
CALL MXV      ( JACOBI, STATE(4), SPHVEL )
```

- After these calls, the vector SPHVEL contains the velocity in spherical coordinates: specifically, the derivatives
- Caution: coordinate transformations often have singularities, so derivatives may not exist everywhere.

(  $d(r) / dt$ ,  $d(\text{colatitude}) / dt$ ,  $d(\text{longitude}) / dt$  )

- » Exceptions are described in the headers of the SPICE Jacobian matrix routines.
- » SPICE Jacobian matrix routines signal errors if asked to perform an invalid computation.

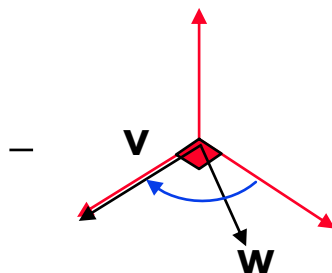
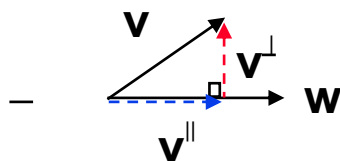


# Vectors

## Navigation and Ancillary Information Facility

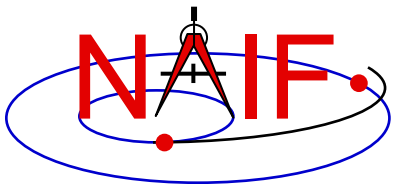
### • Function

- $\langle \mathbf{v}, \mathbf{w} \rangle$
- $\mathbf{v} \times \mathbf{w}$
- $\mathbf{v} / \|\mathbf{v}\|$
- $\mathbf{v} \times \mathbf{w} / \|\mathbf{v} \times \mathbf{w}\|$
- $\mathbf{v} + \mathbf{w}$
- $\mathbf{v} - \mathbf{w}$
- $a\mathbf{v} [+ b\mathbf{w} [+ c\mathbf{u}]]$
- angle between  $\mathbf{v}$  and  $\mathbf{w}$
- $\|\mathbf{v}\|$



### • Routine

- $\text{VDOT}, \quad \text{DVDOT}$
- $\text{VCROSS}, \quad \text{DVCROSS}$
- $\text{VHAT}, \quad \text{DVHAT}$
- $\text{UCROSS}, \quad \text{DUCROSS}$
- $\text{VADD}, \quad \text{VADDG}$
- $\text{VSUB}, \quad \text{VSUBG}$
- $\text{VSCL}, \quad [\text{VLCOM}, \quad [\text{VLCOM3}]]$
- $\text{VSEP}$
- $\text{VNORM}$
- $\text{VPROJ}, \quad \text{VPERP}$
- $\text{TWOVEC}, \quad \text{FRAME}$



# Matrices

Navigation and Ancillary Information Facility

## Selected Matrix-Vector Linear Algebra Routines

- Function

- $M \times v$
- $M \times M$
- $M^t \times v$
- $M^t \times M$
- $M \times M^t$
- $v^t \times M \times v$
- $M^t$
- $M^{-1}$

- Routine

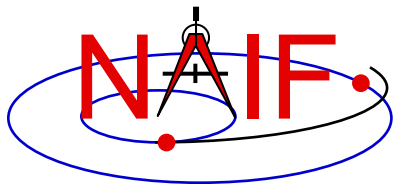
- MXV
- MXM
- MTXV
- MTXM
- MXMT
- VTMV
- XPOSE
- INVERT, INVSTM

**M** = Matrix

**v** = Vector

**x** = Multiplication

**T** = Transpose

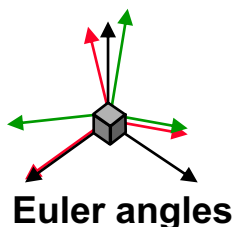


# Matrix Conversions

Navigation and Ancillary Information Facility

## Function

## Routines

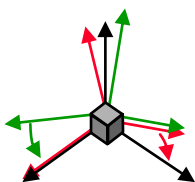


Transform between

$$\begin{matrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{matrix}$$

3x3 rotation matrix

— EUL2M, M2EUL

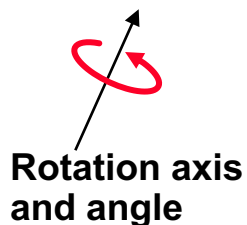


Transform between

$$\begin{matrix} a_x & a_y & a_z & & & \\ b_x & b_y & b_z & & 0 & \\ c_x & c_y & c_z & & & \\ \alpha_x & \alpha_y & \alpha_z & a_x & a_y & a_z \\ \beta_x & \beta_y & \beta_z & b_x & b_y & b_z \\ \gamma_x & \gamma_y & \gamma_z & c_x & c_y & c_z \end{matrix}$$

6x6 state transformation matrix

— EUL2XF, XF2EUL  
RAV2XF, XF2RAV



Transform between

$$\begin{matrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{matrix}$$

3x3 rotation matrix

— RAXISA, AXISAR  
ROTATE, ROTMAT

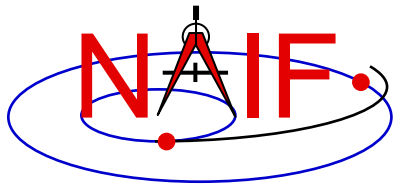
$(Q_0, Q_1, Q_2, Q_3)$   
SPICE Style  
Quaternion

Transform between

$$\begin{matrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{matrix}$$

3x3 rotation matrix

— Q2M, M2Q



# Examples of Computing Derived Quantities

---

Navigation and Ancillary Information Facility

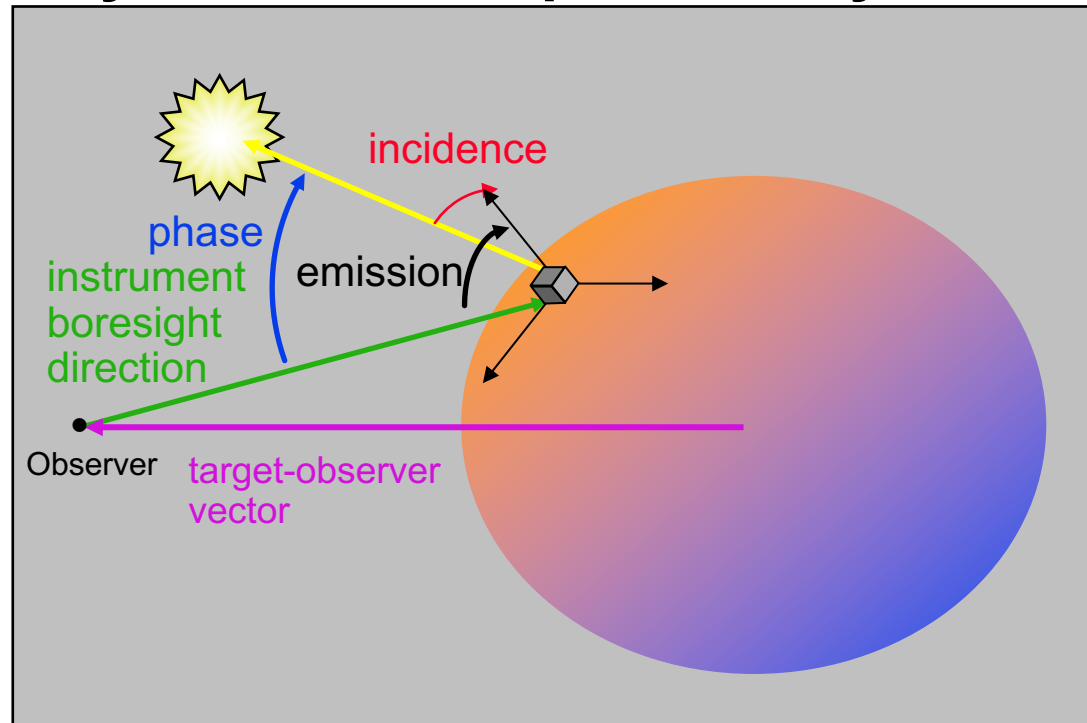
- On the next several pages we present examples of using some of the “derived quantity” APIs.
- Explore the “Most Used SPICE APIs” document to learn more.



# Computing Illumination Angles

Navigation and Ancillary Information Facility

- Given the direction of an instrument boresight in a body-fixed frame, return the illumination angles (incidence, phase, emission) at the boresight's surface intercept on an object, with the object's shape modeled by a tri-axial ellipsoid or by DSK data.

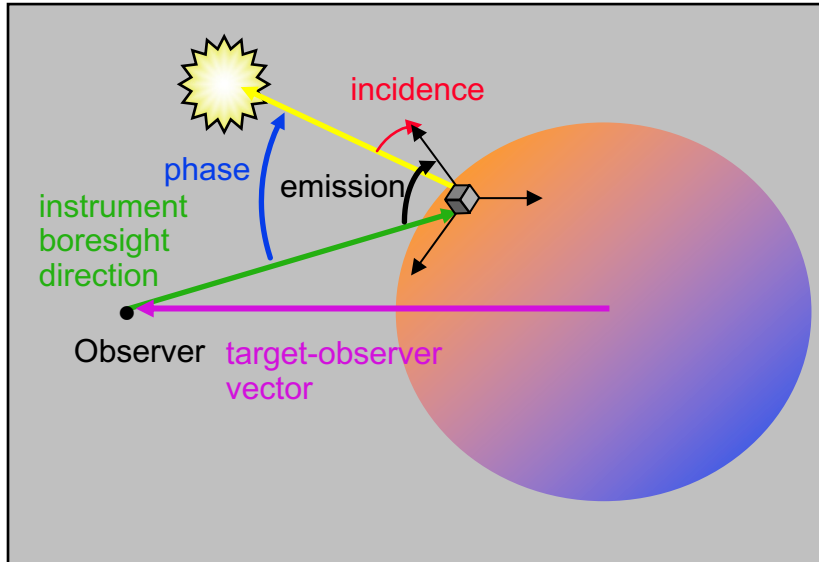




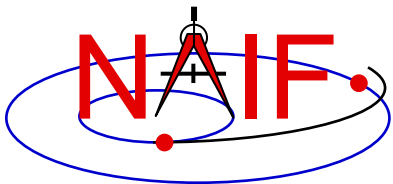


# Computing Illumination Angles

Navigation and Ancillary Information Facility



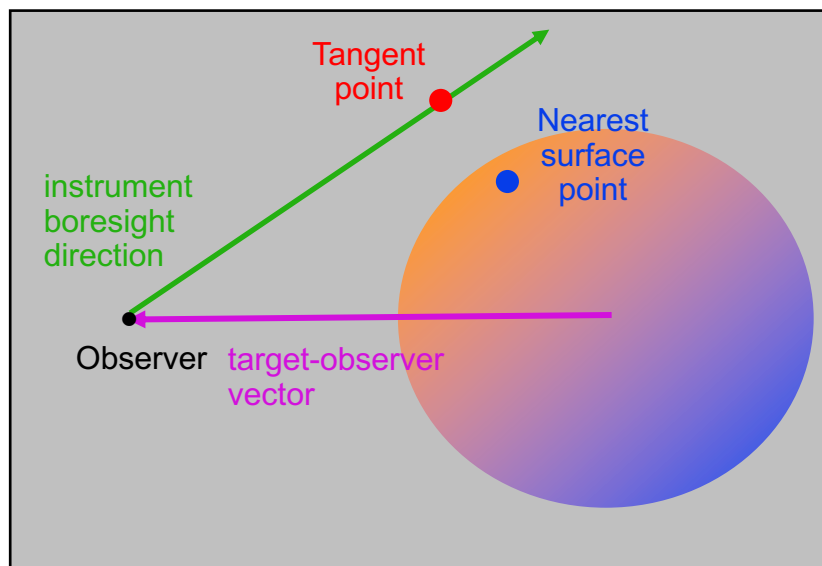
- CALL **GETFOV** to obtain boresight direction vector
- CALL **SINCPT** to find intersection of boresight direction vector with surface
- CALL **ILUMIN** to determine illumination angles



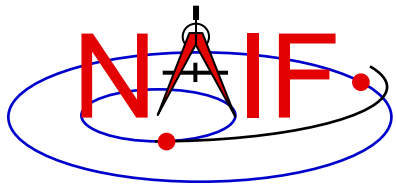
# Computing Tangent Point

Navigation and Ancillary Information Facility

- Given the direction of an instrument boresight, compute the “tangent point” – the point on the boresight ray nearest to the target's surface modeled by a tri-axial ellipsoid.



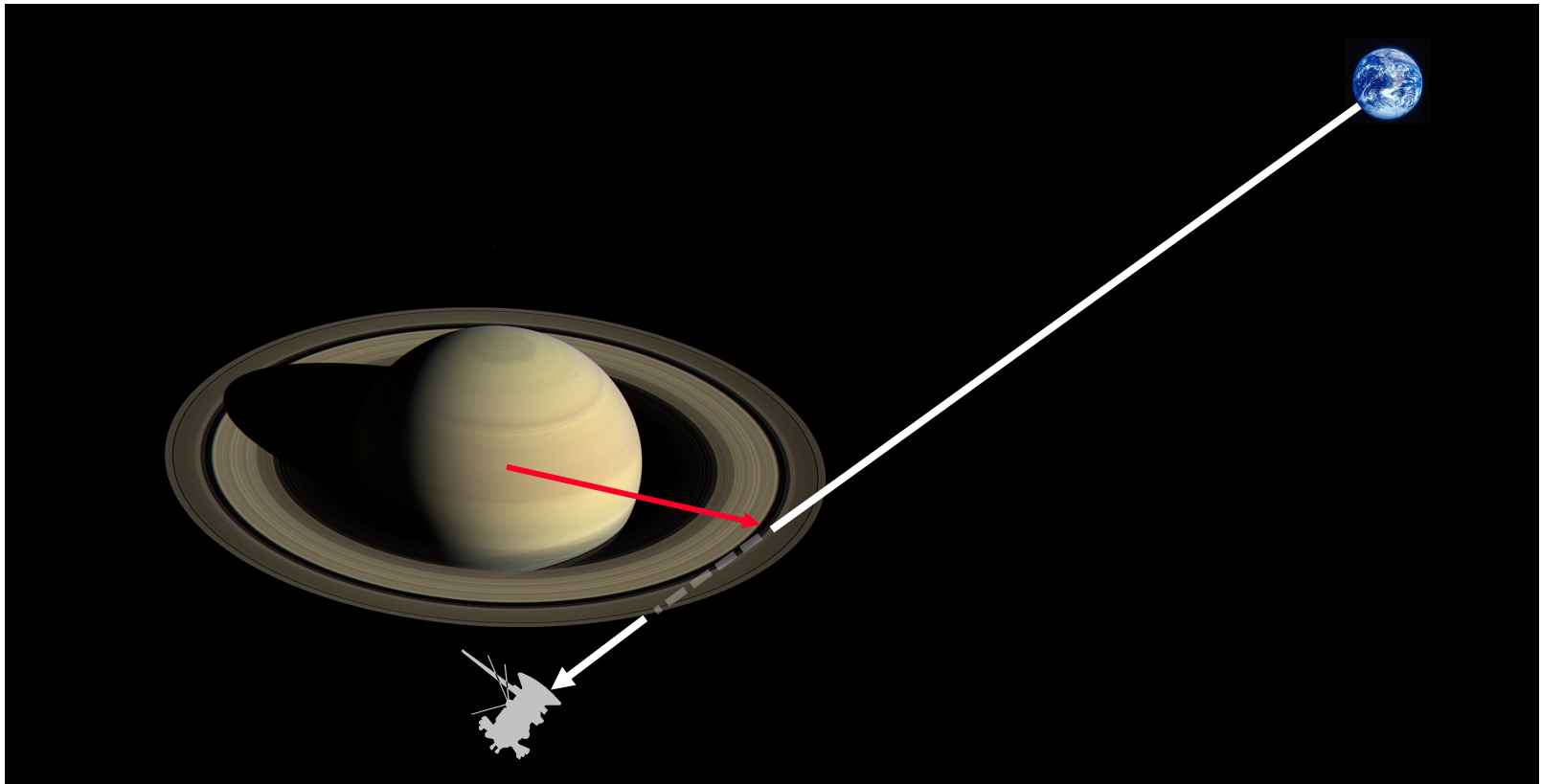
- CALL GETFOV to obtain boresight direction vector
- CALL TANGPT to find the tangent point on the ray and the point on the surface nearest to it

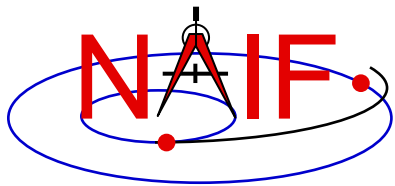


# Computing Ring Plane Intercepts

Navigation and Ancillary Information Facility

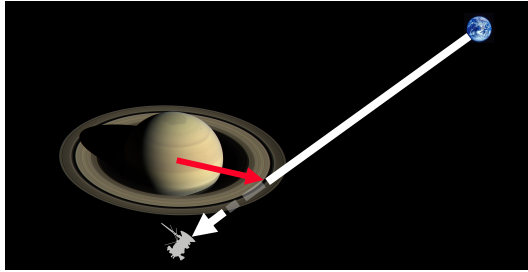
- **Determine the intersection of the apparent line of sight vector between Earth and Cassini with Saturn's ring plane and determine the distance of this point from the center of Saturn.**





# Computing Ring Plane Intercepts-2

Navigation and Ancillary Information Facility

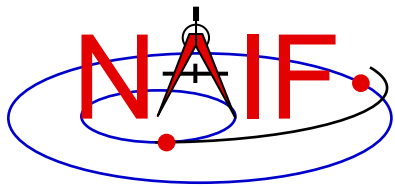


This simplified computation ignores the difference between the light time from Saturn to the Earth and the light time from the ring intercept point to the Earth.

The position and orientation of Saturn could be re-computed using the light time from Earth to the intercept; the intercept could be re-computed until convergence is attained.

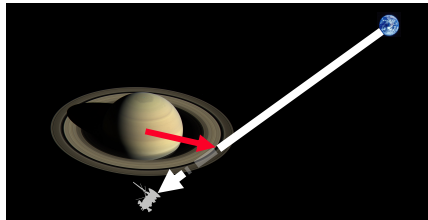
This computation is for the reception case, when radiation is received at the Earth at a given epoch “ET”.

- CALL **SPKEZR** to get light time corrected position of spacecraft as seen from Earth at time ET in J2000 reference frame SCVEC.
- CALL **SPKEZR** to get light time corrected position of center of Saturn at time ET as seen from Earth in J2000 frame SATCTR.
- CALL **PXFORM** to get rotation from Saturn body-fixed coordinates to J2000 at light time corrected epoch. The third column of this matrix gives the pole direction of Saturn in the J2000 frame SATPOL.
- CALL **NVP2PL** and use SATCTR and SATPOL to construct the ring plane RPLANE.
- CALL **INRYPL** to intersect the Earth-spacecraft vector SCVEC with the Saturn ring plane RPLANE to produce the intercept point X.
- CALL **VSUB** to get the position of the intercept with respect to Saturn XSAT (subtract SATCTR from X) and use **VNORM** to get the distance of XSAT from the center of Saturn.



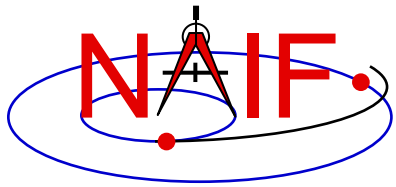
# Computing Ring Plane Intercepts-3

## Navigation and Ancillary Information Facility



An  
alternate  
approach

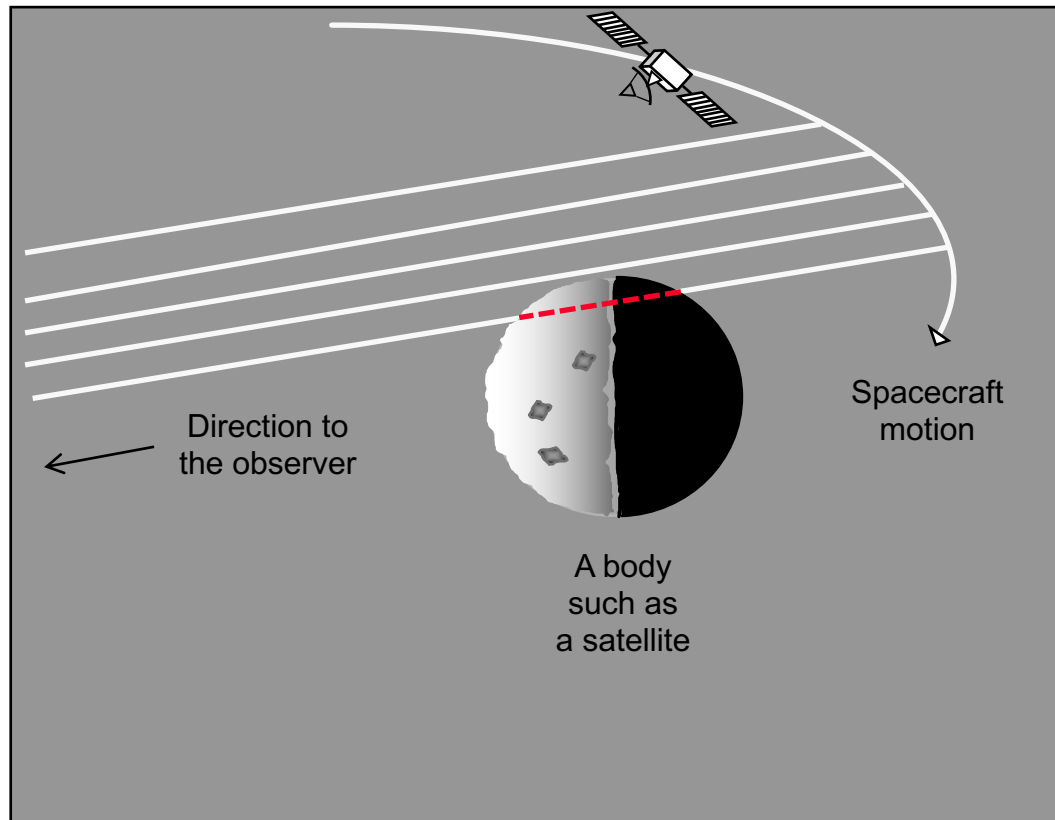
- Create a dynamic frame with one axis pointing from Earth to the light time corrected position of the Cassini orbiter. Use the CN correction for this position vector. (This gives us a frame in which the direction vector of interest is constant.)
- Temporarily change the radii of Saturn to make the polar axis length 1 cm and the equatorial radii 1.e6 km. This can be done either by editing the PCK or by calling **BODVCD** to fetch the original radii, then calling **PDPOOL** to set the kernel pool variable containing the radii to the new values. This flat ellipsoid will be used to represent the ring plane.
- Use **SINCPT** to find the intercept of the Earth-Cassini ray with the flat ellipsoid. Use the CN correction. SINCPT returns both the intercept in the IAU\_SATURN frame and the Earth-intercept vector. Use **VNORM** to get the distance of the intercept from Saturn's center.
- Restore the original radii of Saturn. If PDPOOL was used to update the radii in the kernel pool, use **PDPOOL** again to restore the radii fetched by BODVCD.



# Computing Occultation Events

Navigation and Ancillary Information Facility

- **Determine when the spacecraft will be occulted by an object (such as a natural satellite) as seen from an observer (such as Earth).**





# Find Occultation Ingress/Egress

## Navigation and Ancillary Information Facility

- **Select a start epoch, stop epoch and step size.**
  - Start and stop epochs can bracket multiple occultation events
  - Step size should be smaller than the shortest occultation duration of interest, and smaller than the minimum interval between occultation events that are to be distinguished, but large enough to solve the problem with reasonable speed.
- **Insert search interval into a SPICE window. This is the “confinement window.”**
- **CALL GFOCLT to find occultations, if any.** The time intervals, within the confinement window, over which occultations occur will be returned in a SPICE window.
  - GFOCLT can treat targets as ellipsoids, DSK shapes, or points (but at least one must be an ellipsoid or DSK shape, and DSK shapes can be used only with point targets).
  - GFOCLT can search for different occultation or transit geometries: full, partial, annular, or “any.”

